

Roles of quark-pair correlations in baryon structure and non-leptonic weak transitions of hyperon

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Abstract

Roles of quark-pair correlations in the baryon structure and the hyperon non-leptonic weak decay are studied within the non-relativistic constituent quark model. We construct the SU(3) ground state baryons by solving the three body problem rigorously with the confinement force and the short range spin-dependent attraction. We emphasize the importance of the $s = 0$ quark-quark correlation to reproduce the $\Delta I = 1/2$ enhancement of the hyperon decay, and demonstrate that resulting static properties as well as the decay amplitudes agree with the experiments, if we deal with the $s = 0$ correlation properly. Special attention is also put on the consequences of the SU(6) spin-flavor symmetry breaking due to the $s = 0$ correlation. Calculated magnetic moments are the almost same as the naive SU(6) predictions in spite of the existence of the strong correlations.

1 Introduction

Properties of light baryons have been extensively studied by various models based on the constituent quark picture. Their results are consistent with experiments including applications for the one and two nucleon systems, although this model involves several adjustable parameters. Despite the success of this approach, it is obscure whether or not these models correctly describe the quark distributions in the baryons, and in fact understandings of the non-leptonic weak hyperon decay and its $\Delta I = 1/2$ rule are still incomplete.

The $\Delta I = 1/2$ rule implies dominance of the $\Delta I = 1/2$ transitions on the non-leptonic hyperon decay[1]. It is known that relative magnitudes of parity conserving decay amplitudes are reasonably described within the baryon pole approximation, where the weak decay takes place as the two quark transition process: a us -pair in the initial hyperon with their total spin s being 0 changes to a ud -pair in the final state baryon. However, if one calculates them using the constituent quark models, the absolute value of the amplitudes is about a half of the experimental data at most[1, 2]. We emphasize

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here, because of the heavy W -boson mass, the weak matrix elements are quite sensitive to the short range quark-quark correlations[3]. Consequently, the failure of the constituent quark model to describe the non-leptonic weak transition may indicate the lack of the quark correlation in the $s = 0$ channel.

On the other hand, it was pointed out that, from both theoretical and phenomenological points of view[4], there exists a strong correlation between quarks in the $s = 0$ channel.

In this work we try to clarify the roles of the quark correlation in the baryon structure and hyperon non-leptonic weak decays by using the constituent quark model. We assume the short range spin-dependent correlations between quarks together with the confinement force, and calculate the baryon masses and other static properties. In order to deal with the spin-dependent correlation correctly, we must rigorously solve the three body problem. For this purpose, we adopt the coupled-rearrangement-channel variational method with the Gaussian basis functions which has been developed by Hiyama and Kamimura[5].

We also focus on the SU(6) breaking effects on baryon properties. Introduction of the spin-dependent correlation naturally spoils the SU(6) spin-flavor symmetry which is known to work well for e.g. the baryon magnetic moments. We shall calculate the magnetic moments to estimate the SU(6) breaking effects clearly.

2 Calculation of weak decay matrix elements

Let us write the low energy effective weak interaction Hamiltonian[6];

$$\mathcal{H}_W = \frac{G_F \sin \theta \cos \theta}{\sqrt{2}} \sum_i c_i(\mu^2) O_i + \text{h.c.} \quad (1)$$

where O_i are the quark 4-Fermi operators, and O_1, O_2 give dominant contributions in our case. We take values of c_i given in ref. [6] at 1GeV^2 .

Our task here is to evaluate the matrix element $\langle B_f \pi^a | \mathcal{H}_W | B_i \rangle$ for the strangeness changing process $B_i \rightarrow B_f + \pi^a$. The PCAC relation and soft pion theorem are suitable to deal with the strong interacting pion-nucleon system. Using them, one can find the baryon pole formula to the parity conserving amplitudes. For example, $\Lambda^0 \rightarrow n + \pi^0$ pole amplitude is given by,

$$\frac{M_N + M_\Lambda}{f_\pi} \left[G_{nn}^{\pi^0} \frac{1}{M_\Lambda - M_N} \langle n | \mathcal{H}_W | \Lambda \rangle + \langle n | \mathcal{H}_W | \Sigma^0 \rangle \frac{1}{M_N - M_\Sigma} G_{\Lambda\Sigma}^{\pi^0} \right] \quad (2)$$

where $\langle n | \mathcal{H}_W | \Lambda \rangle$ and $\langle n | \mathcal{H}_W | \Sigma^0 \rangle$ are the matrix elements of eq. (1) with appropriate baryon states, and $G_{B B'}^{\pi^a}$ denote the axial vector coupling constants which gives probabilities for the pion emission $B \rightarrow B' + \pi^a$ and are constrained by experiments.

We come to determine the matrix elements of \mathcal{H}_W , $\langle n | \mathcal{H}_W | \Lambda \rangle$ and $\langle p | \mathcal{H}_W | \Sigma^+ \rangle$. We recall the quark models such as Harmonic Oscillator model or MIT bag model give much smaller values for these matrix elements than the data[2]. It is instructive to rewrite the $V - A$ operators O_1, O_2 in the non-relativistic limit in the coordinate space as

$$O_1, O_2 \rightarrow a_d^\dagger a_u^\dagger (1 - \vec{\sigma}_u \cdot \vec{\sigma}_s) \delta^{(3)}(\vec{r}_{us}) a_u a_s \quad (3)$$

where a_i, a_i^\dagger are annihilation and creation operators of quarks with flavor i . Presence of the spin-projection operator $(1 - \vec{\sigma}_u \cdot \vec{\sigma}_s)$ tells us that the weak transition is generated by the two body process between spin-0 quark pairs; $(us)^0 \rightarrow (ud)^0$, which guarantees the $\Delta I = 1/2$ dominance on the non-leptonic hyperon decays due to the antisymmetrization of the quark-pairs. Now it is clear that this decay amplitude is sensitive to the correlation of the spin-0 quark pair in the baryons. The standard constituent quark model never incorporates such a quark-quark correlation properly.

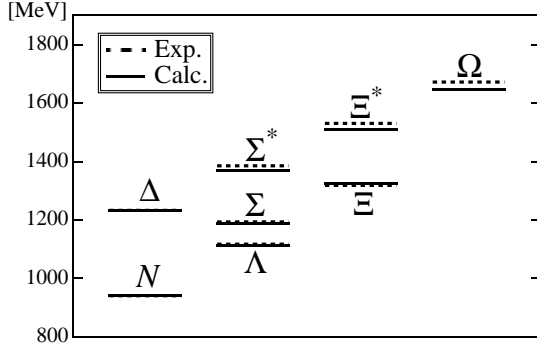


Figure 1: SU(3) baryon mass spectrum

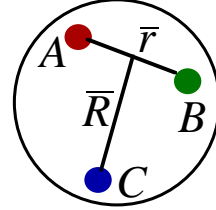


Figure 2: Configuration of three quarks in nucleon

3 Constituent quark model with the spin-dependent correlations

We phenomenologically introduce the effective Hamiltonian which includes the confinement force and the spin-dependent part as;

$$\mathcal{H} = \sum_i \frac{p_i^2}{2m_i} + \sum_{i < j} \frac{1}{2} K (\vec{r}_i - \vec{r}_j)^2 + \sum_{i < j} V_S(ij) + V_0 \quad (4)$$

$$V_S(ij) = \begin{cases} 0 & (s = 1 \text{ pair}) \\ \frac{C_{SS}}{m_i m_j} \text{Exp} \left[-(\vec{r}_i - \vec{r}_j)^2 / \beta^2 \right] & (s = 0 \text{ pair}) \end{cases} \quad (5)$$

where m_i are the constituent quark masses, and K, C_{SS}, β are the model parameters. V_0 contributes to the over all shift of the resulting spectrum and is chosen to adjust the energy of the lowest state to the nucleon mass. Constituent quark masses are taken to be $m_u = m_d = 330\text{MeV}$ and $m_s = 510\text{MeV}$.

Using this Hamiltonian, we shall solve non-relativistic three body problem rigorously. We use the coupled-rearrangement-channel variational method with Gaussian basis functions[5]. We assume the isospin symmetry between u and d quarks, and solve the three body problem without further approximations or assumptions. The strange quark is explicitly distinguished from light u, d quarks.

We shall fix the model parameters so as to reproduce the nucleon and Δ masses, proton radius and the $\Sigma^+ \rightarrow n\pi^+$ amplitude, since this decay is completely dominated by the baryon pole diagrams (see Table 1). We obtain the parameters $K = 0.005\text{GeV}^3$, $\beta = 0.5\text{fm}$ and $C_{SS}/m_u^2 = 1.4\text{GeV}$.

Resulting mass spectrum is shown in Fig.1. It could be possible to obtain a better agreement by modifying the potential or parameters, but the present results are enough for our purpose. The effect of the attractive correlation can be seen clearly by introducing the average distances of the Jacobi coordinate \bar{r} and \bar{R} defined in Fig.2, where all possible rearrangement channels are transformed to the configuration of Fig.2. We find, for the nucleon, $\bar{r} = 0.92\text{fm}$ and $\bar{R} = 0.97\text{fm}$ when the total spin s of the quark pair A and B is zero, while $\bar{r} = 1.1\text{fm}$ and $\bar{R} = 0.81\text{fm}$ in the $s = 1$ case. Apparently, the quark correlation modifies quark distribution in the nucleon. The value of the wave function at origin $\int d^3R d\Omega_r \Psi(r=0, R)$ in the $s = t = 0$ case is about three times as large as that of the $s = t = 1$ case, which provides an huge enhancement for the $\Delta I = 1/2$ weak decay amplitudes.

The matrix elements of the weak Hamiltonian are calculated in terms of our wave functions. We find $\langle n | \mathcal{H}_W | \Lambda \rangle = -0.946 (\times 10^{-2} \text{GeV}^3)$ as the full result and -0.310 when we omit the correlation. Similarly, $\langle p | \mathcal{H}_W | \Sigma^+ \rangle = 2.86$ with V_S , while it becomes 0.762 without V_S . In the absence of the correlation V_S , a ratio $\langle p | H_W | \Sigma^+ \rangle / \langle n | H_W | \Lambda \rangle = -2.45$ shows a perfect agreement with the SU(6) expectation $\sqrt{6} \simeq -2.4494 \dots$. In the realistic case with V_S , one can observe the substantial enhancement and the SU(6) breaking effect.

With these values, calculated weak transition amplitudes are tabulated in Table 1. We show the pole contributions only in the second column, and the sum of the pole, factorization and penguin contributions in the third column to be compared with the experiments. We find a good agreement for $\Sigma \rightarrow N\pi$ decays, while the $\Lambda \rightarrow N\pi$ amplitude is not enough. We note that the small value of the $\Lambda \rightarrow N\pi$ pole contribution is caused by the strong cancellation of the two terms in eq. (2). If we vary the axial-vector coupling G within the experimental errors ($\sim 10\%$), we find about 40% increase of the $\Lambda \rightarrow N\pi$ pole contribution with the Σ decay amplitudes almost unchanged.

4 SU(6) symmetry breaking effects on the magnetic moment

Our wave functions clearly violate the naive SU(6) spin-flavor symmetry for the baryon. In fact, the ratio $\langle p | H_W | \Sigma^+ \rangle / \langle n | H_W | \Lambda \rangle$ becomes -3.02 , instead of the SU(6) value -2.45 . Thus, we estimate the size of the SU(6) breaking effect to be about 20%, which is significant. On the other hand, it is historically known that the light baryon magnetic moments are well reproduced in the naive quark model by virtue of the SU(6) spin-flavor symmetry. Hence, it is important to examine the SU(6) breaking by calculating the magnetic moments.

Our results are shown in Table 2. It is manifest that the results are almost unchanged even after introducing the spin-dependent correlations. The differences are of order of a few % in any cases. It seems that the global baryon properties such as magnetic moments obtained by integrating the wave function over space are insensitive to the quark correlation, although the local structure of the quark wave function is modified substantially.

Table 1 Parity conserving weak transition amplitude (in 10^{-7} unit)

	Pole	total	Exp.
Σ_0^+	24.0	26.1	26.24
Σ_+^+	43.3	43.3	41.83
Λ_0^0	-3.82	-8.84	-15.61

Table 2 Magnetic Moments

	full	no V_S	Exp.
μ_p	2.75	2.84	2.79
μ_n	-1.78	-1.90	-1.91
μ_Λ	-0.60	-0.61	-0.61
μ_{Σ^+}	2.67	2.73	2.46
μ_{Σ^-}	-1.05	-1.06	-1.16
μ_{Ξ^0}	-1.40	-1.45	-1.25

5 Summary

In conclusion, we have studied the roles of the spin-dependent quark correlations in the baryon structure. We have emphasized that the non-leptonic weak transition of the hyperon is unique quantity to investigate the quark-quark correlation in the spin-0 channel. We have solved three body problem explicitly using the coupled-rearrangement-channel variational method. Results for static baryon properties as well as the transition amplitudes of the non-leptonic hyperon decay reasonably agree with the empirical values. We have also discussed the SU(6) breaking effects on the baryon properties, and pointed out that this symmetry is still useful for the static baryon properties, although local behavior of the quark wave function considerably departs from the SU(6) expectation.

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